1.

Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

a) Find $P(A'|B')$

$$\Rightarrow P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{0.33}{0.55} = 0.6 \quad ①$$

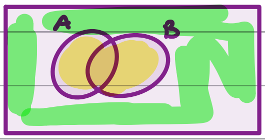
$$\Rightarrow \underline{P(A'|B')} = 0.6 \quad ①$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(B') = 1 - 0.45 = 0.55$

$P(B) = 0.45$

$P(A' \cap B') \Rightarrow$



$P(A \cup B) = 0.35 + 0.45 - 0.13 = 0.67$

$\Rightarrow P(A' \cap B') = 1 - 0.67 = 0.33$

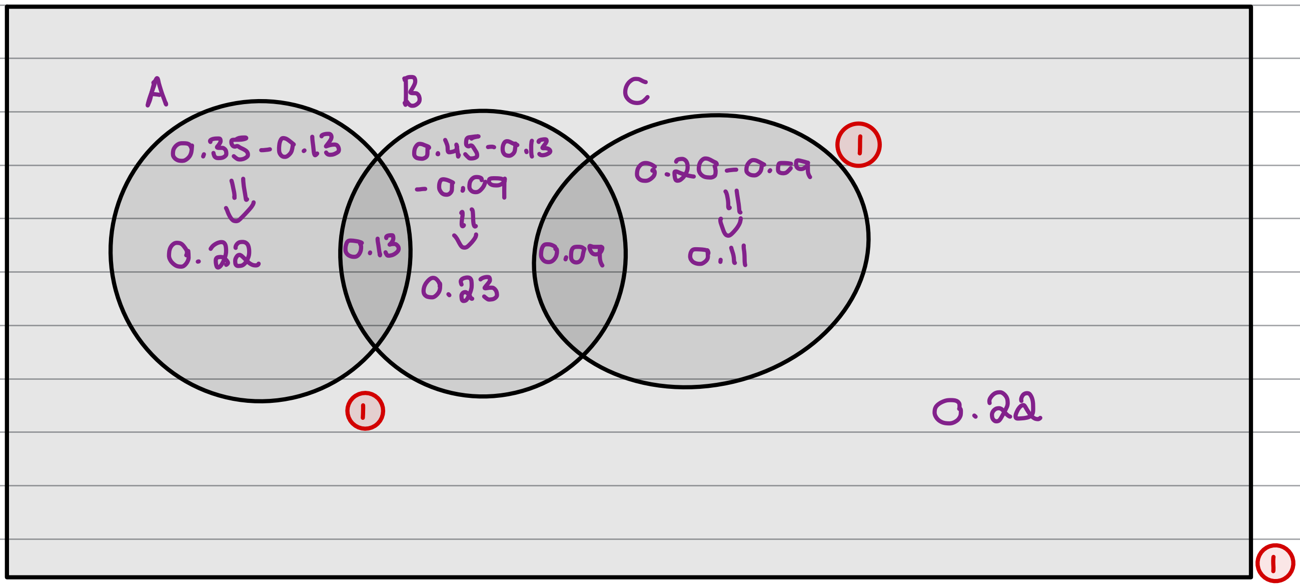
b) If the event A and the event B are independent, then $P(A) \cdot P(B) = P(A \cap B)$. So, for our case we have the required information, so we can compute it to see if they are independent.

$\Rightarrow P(A \cap B) = 0.13$, $P(A) = 0.35$, $P(B) = 0.45$ $\Rightarrow P(A) \cdot P(B) = 0.35 \times 0.45$
$= 0.1575 \neq 0.13$

$\Rightarrow$ Since $P(A \cap B) \neq P(A) \cdot P(B)$, A and B are <u>not</u> independent. ①

c) • Mutually exclusive => two events cannot happen Similtaneously => A and C cannot happen Similtaneously.

• Statistically independent = independent => B and C are independent.



$P(A) = 0.35$, $P(B) = 0.45$, $P(A \cap B) = 0.13$, $P(C) = 0.20$

B and C are independent, hence $P(B \cap C)$ ① $P(B)P(C) = 0.45 \times 0.20 = 0.09$ ①

$P(A \cup B \cup C) = 1 - (0.22 + 0.13 + 0.23 + 0.09 + 0.11) = 1 - 0.78 = 0.22$

d) $P([B \cup C]')$ will be everthing that is not part of B and/or C.

$\Rightarrow P([B \cup C]') = 0.22 + \overset{①}{0.22} = \underline{\underline{0.44}}$ ①

2.

Three bags, A, B and C, each contain 1 red marble and some green marbles.

Bag A contains 1 red marble and 9 green marbles only $1+9=10$
Bag B contains 1 red marble and 4 green marbles only $1+4=5$
Bag C contains 1 red marble and 2 green marbles only $1+2=3$

Sasha selects at random one marble from bag A.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag B.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag C.

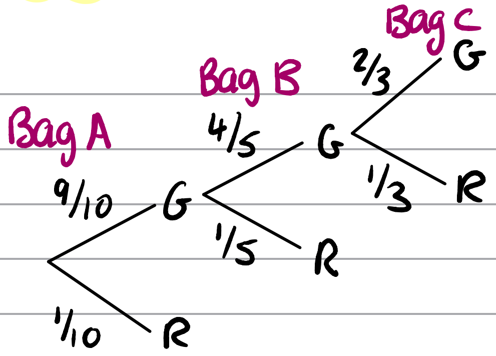
(a) Draw a tree diagram to represent this information. **(2)**

(b) Find the probability that Sasha selects 3 green marbles. **(2)**

(c) Find the probability that Sasha selects at least 1 marble of each colour. **(2)**

(d) Given that Sasha selects a red marble, find the probability that he selects it from bag B. **(2)**

a)



②

b) For "And" use multiplication

$P(3 \text{ green marbles}) = \frac{9}{10} \times \frac{4}{5} \times \frac{2}{3} = \frac{12}{25}$ ①

①

c)

$P(\text{at least } 1 \text{ marble of each colour})$

$= \left(\frac{9}{10} \times \frac{1}{5}\right) + \left(\frac{9}{10} \times \frac{4}{5} \times \frac{1}{3}\right) = \frac{21}{50}$ ①

①

For "Or" use addition

d)

'given that'

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

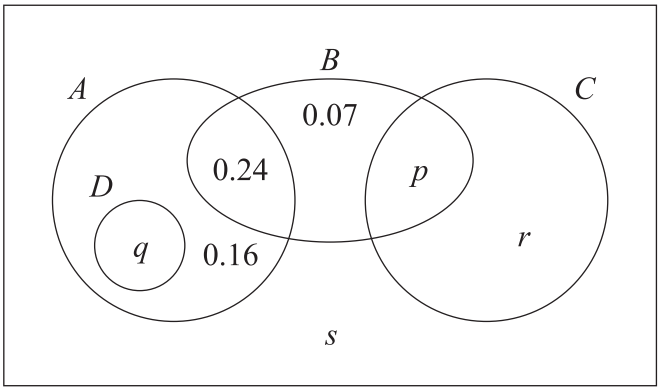
$$P(\underset{E}{\text{Red from B}} \mid \underset{F}{\text{Red selected}})$$

$$= \frac{\frac{9}{10} \times \frac{1}{5}}{\left(\frac{1}{10}\right) + \left(\frac{9}{10} \times \frac{1}{5}\right) + \left(\frac{9}{10} \times \frac{4}{5} \times \frac{1}{3}\right)}$$ ①

$$= \frac{\frac{9}{50}}{\frac{13}{25}} = \frac{9}{26}$$ ①

3.

The Venn diagram shows the probabilities associated with four events, A, B, C and D



(a) Write down any pair of mutually exclusive events from A, B, C and D

Event 1 and event 2 are mutually exclusive if they can't happen at the same time

(1)

Given that $P(B) = 0.4$

(b) find the value of p

(1)

Given also that A and B are independent

(c) find the value of q

(2)

Given further that $P(B' | C) = 0.64$

(d) find

(i) the value of r

(ii) the value of s

(4)

a) A and C or D and C or D and B ① For any of those pairs

b) $0.24 + 0.07 + p = 0.4$

$p = 0.4 - 0.24 - 0.07$

$= 0.09$ ①

c) If A and B are independant P(A and B) = P(A) × P(B)

$P(A \text{ and } B) = 0.24$

$P(B) = 0.4$

$\therefore 0.24 = P(A) \times 0.4$ ①

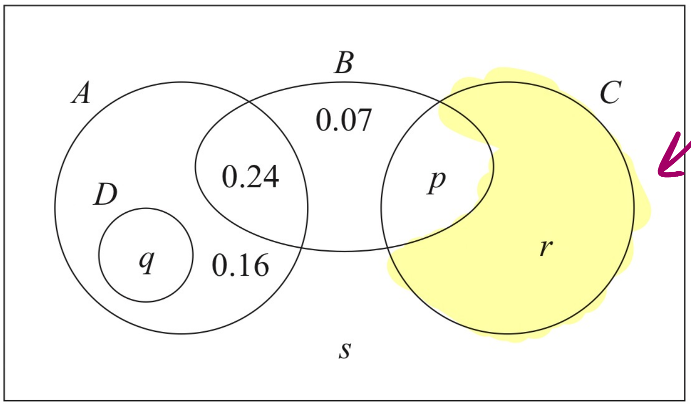
$P(A) = 0.6$

$0.24 + 0.16 + q = 0.6$

$q = 0.6 - 0.24 - 0.16$

$= 0.20$ ①

The Venn diagram shows the probabilities associated with four events, A, B, C and D



shaded area
$P(C \cap B')$

d) $P(B' \mid C) = \frac{P(C \cap B')}{P(C)} = \frac{r}{p+r}$ ①

i)

$\therefore\ 0.64 = \frac{r}{p+r}$ since $p = 0.09$ $0.64 = \frac{r}{0.09 + r}$

$0.64(0.09 + r) = r \Rightarrow 0.0576 + 0.64r = r$

$\Rightarrow 0.36r = 0.0576 \Rightarrow r = 0.16$ ①

ii) $0.4 + 0.16 + 0.36 + s = 1$ ①

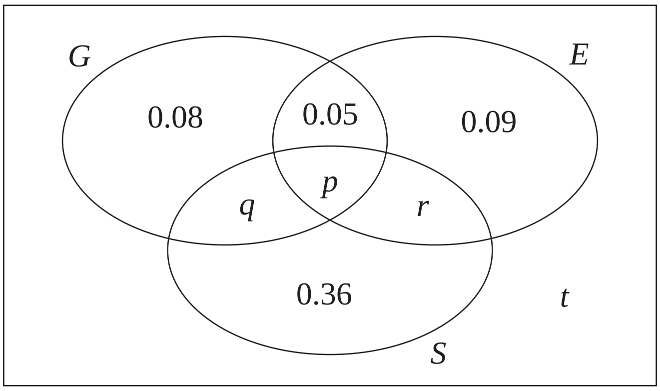
$s = 1 - 0.4 - 0.16 - 0.36$

$= 0.08$ ①

4. A large college produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the college is selected at random and the events G, E and S are defined as follows

G is the event that the student reads the magazine about green issues
E is the event that the student reads the magazine about equality
S is the event that the student reads the magazine about sports

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines.

(1)

so $P(G \cap E \cap S) = 0$

No students read all three magazines and $P(G) = 0.25$

(b) Find

(i) the value of p

(ii) the value of q

(3)

Given that $P(S \mid E) = \frac{5}{12}$

(c) find

(i) the value of r

(ii) the value of t

(4)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.
Show your working clearly.

(3)

a) 'exactly one' so use OR rule of addition

$0.08 + 0.09 + 0.36 = 0.53$ ①

4.

b) i) $P(G \cap E \cap S) = 0 \implies p = 0$ ①

ii) $0.08 + 0.05 + q + p = 0.25$ because $p = 0$ ①

$0.08 + 0.05 + q = 0.25$

$q = 0.12$ ①

values for conditional probability read from diagram.

c) i) $$\frac{r + p}{r + p + 0.09 + 0.05} = \frac{5}{12}$$ ①

$$\frac{r}{r + 0.14} = \frac{5}{12}$$

$12r = 5(r + 0.14)$ ①

$12r = 5r + 0.7$

$7r = 0.70$

$r = 0.10$ (2.s.f) ①

ii) $0.08 + 0.05 + \underset{(q)}{0.12} + \underset{(p)}{0} + 0.09 + \underset{(r)}{0.10} + 0.36 + t = 1$

$0.8 + t = 1$

$\therefore t = 0.2$ ①

d) $P(S \cap E') = 0.36 + \underset{(q)}{0.12}$

$= 0.48$ ①

$P([(S \cap E')] \cap G) = \underset{(q)}{0.12}$

$P(G) = 0.25$ so $P(S \cap E') \times P(G) = 0.48 \times 0.25$ ①

$= 0.12$

$P(S \cap E') \times P(G) = P([(S \cap E')] \cap G) = 0.12$

$\therefore$ they are independant. ①

5. The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1% of females from this country are shorter than k cm,

↪ 0·01

(a) find the value of k

(2)

(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

$P(150 < F < 175)$

(1)

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm

(c) Find the probability that her height is more than 160 cm

(4)

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm $H_1 : \mu < 166{\cdot}5$

Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm

test stat $n = 50$

(d) Carry out a suitable test to assess Mia's belief.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

a) $F \sim N(166{\cdot}5, 6{\cdot}1^2)$ where $\mu = 166.5$, $\sigma = 6{\cdot}1$

$P(F < k) = 0{\cdot}01$

$\frac{k - 166{\cdot}5}{6{\cdot}1} = -2{\cdot}3263$ ① this value comes from the z normal distribution with $\mu = 0$, $\sigma = 1$

$\frac{k - \mu}{\sigma} = z$

$k - 166{\cdot}5 = -2{\cdot}3263 \times 6{\cdot}1$

$k = -14{\cdot}19043 + 166{\cdot}5$

$k = 152{\cdot}309..$

$k = 152$ (3 s.f) ①

b) $P(150 < F < 175) = 0{\cdot}91484...$

$= 0{\cdot}915$ (3.s f) ①

use 'Normal CD' function on calculator, LB = 150, UB = 175.

c) $P(F > 160 \mid 150 < F < 175)$ ① $= \dfrac{P(160 < F < 175)}{P(150 < F < 175)}$ ①

$P(F > 160)$ given that $(150 < F < 175)$

$= \dfrac{P(160 < F < 175)}{0.915}$ ← from b)

$= \dfrac{0.77494\ldots}{0.915}$ ①

$= 0.847$ (3.s.f) ①

d) Conducting a normal hypothesis test because we are looking at changes to μ.

$H_0 : \mu = 166.5$ H_1 $\mu < 166.5$ ① ← testing if the mean has decreased.

5% level of significance ⟹ C.V = 0.05

Let X = height of female from 2nd country

$\bar{X} \sim N\left(166.5, \left(\frac{7.4}{\sqrt{50}}\right)^2\right)$ ①

$\sigma_2 = \left(\frac{\sigma_1}{\sqrt{n}}\right)^2$

$P(\bar{X} < 164.6) = 0.03472..$ ①

$0.03472... < \underset{(c.v)}{0.05}$ ∴ value is significant, reject Ho

There is sufficient evidence to support Mia's belief. ①